# THEORETICAL AND EXPERIMENTAL ANALYSIS OF FERRITE CIRCULAR RESONATORS IN NONRADIATIVE DIELECTRIC STRUCTURES

**Stefano Alei, Carlo Di Nallo, Maurizio Fascetti, Fabrizio Frezza, Alessandro Galli, and Giorgio Gerosa**

Electronic Engineering Department - "La Sapienza" University of Rome, Italy

**Abstract**

A complete characterization of cylindrical resonators made of ferrite is presented concerning nonradiative dielectric (NRD) components, for promising applications to tunable filters, circulators, etc., at microwaves and millimeter waves. For such class of anisotropic resonators, the frequency spectrum and the relevant field configurations are accurately derived for any resonance mode excitable in NRD topology, as a function of the geometric and electromagnetic parameters. In particular, the tuning and nonreciprocal selective properties are investigated as the bias magnetization is varied. The dissipation effects are also quantified rigorously, by calculating with closed-form approaches both the complex resonant frequencies and the quality factors due to losses in the ferrite material and in the metal plates. All these theoretical results have been discussed and validated also experimentally by means of various measurements on NRD prototypes.

## 1. Introduction

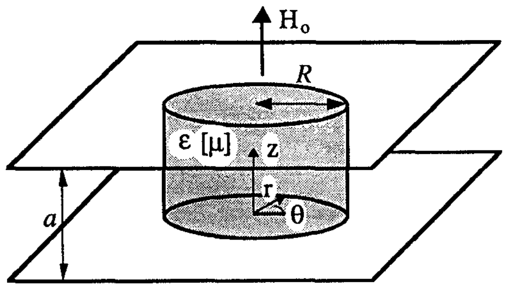
Various types of anisotropic resonators are commonly employed at microwaves in different circuit topologies (metallic waveguides, strip line, microstrip, etc.) for nonreciprocal devices, such as circulators or tunable filters, and for measurements of the electromagnetic properties of materials as well [1].

The goal of this work is to investigate in a complete and accurate way the characteristics and the potentialities offered by ferrite resonators referring to the 'nonradiative dielectric' (NRD) circuitry [2], which represents a rather recent but very promising alternative to other common technologies for communication systems, particularly at millimeter waves.

As is known, the basic NRD devices are essentially constituted by field-confining dielectric components, such as waveguides and resonators, that are sandwiched between wide metal parallel plates placed at a distance apart less than half a free-space wavelength. The consequent important advantageous feature of such structures is represented by strong reduction of interference, radiation, and loss effects, related to discontinuities, bends, etc., present in integrated circuits [2,3].

A variety of NRD devices and subsystems has been proposed and tested in practice. In particular, resonant devices of isotropic type have already been extensively analyzed by using straightforward topologies with dielectric resonators [2,4]. Nevertheless, till now the potentialities of anisotropic NRD resonators have not yet been analyzed in detail (as far as we know, just a practical example of a NRD ferrite circulator was proposed in the literature [5]).

The use of anisotropic resonators further enriches the operational functions that can be achieved with NRD circuitry. The devices derivable from the NRD ferrite resonators, here analyzed both from a theoretical and from a practical viewpoint, possess specific advantageous features related to its circuit topology (e.g., the possibility of simple circuitry integrations and the rigorous characterization of their electromagnetic behavior). The present analysis intends to provide all the fundamental information required to accurately predict the performances of such NRD components and also to suggest novel possibilities for advanced nonreciprocal integrated devices.




$$[\mu] = \mu_o[\mu_r] = \mu_o \begin{bmatrix} \mu_1 & j\mu_2 & 0 \\ -j\mu_2 & \mu_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{cases} \mu_1 = 1 + \dfrac{\omega_m \omega_o}{\omega_o^2 - \omega^2} \\ \mu_2 = \dfrac{\omega\, \omega_m}{\omega_o^2 - \omega^2} \end{cases} \quad \omega_o = -\gamma H_o, \; \omega_m = -\frac{\gamma}{\mu_o} M_o, \; \gamma\text{, gyromagnetic ratio}$$

**Fig. 1** - A cylindrical circular-section ferrite resonator (FR) in nonradiative dielectric (NRD) configuration: coordinate system $(r,\theta,z)$, geometrical parameters (height $a$, radius $R$) and physical parameters (permittivity $\varepsilon$ and permeability tensor $[\mu]$, whose elements are given as a function of a $z$-oriented bias saturation field $H_o$, of magnetization $M_o$, and of the angular frequency $\omega=2\pi f$). *Parameters of the NRD FR usually considered in our theoretical and experimental simulations (if not specified otherwise):* $a=12.3$ *mm;* $R=8.175$ *mm;* $\varepsilon=\varepsilon_o\varepsilon_r=16\varepsilon_o$*;* $M_o=1780$ *gauss.*





0-7803-3246-6/96/$5.00 © IEEE

1996 IEEE MTT-S Digest

## 2. Theoretical analysis of cylindrical NRD ferrite resonators

The basic NRD structure under investigation consists of a cylindrical circular-section (usually called 'disk' or 'pillbox') ferrite resonator (here also shortened with the acronym 'FR'), inserted between two parallel metal plates, as schematized in Fig. 1, with the related electromagnetic and geometric parameters, the coordinate system $(r,\theta,z)$, etc.

The ferrite rod is supposed to be biased by a vertical ($z$) static magnetic field intensity $H_o$, and presents the typical permeability tensor with the matrix elements having the well-known dispersive expressions [1], as resumed in Fig. 1 as well.

First, we will focus our attention on the electromagnetic behavior of the ideal lossless structure (i.e., metal plates with a conductivity $\sigma=\infty$ and ferrite with a damping factor $\alpha=0$). It should be pointed out that in this case the electromagnetic analysis of disk FRs is relatively straightforward for NRD topology, since the resonances can be derived with a simple extension of a known approach used for ferrite-filled circular waveguides [6].

In our case, the longitudinal field dependence is fixed by the presence of the metal plates which discretizes the vertical wavenumber as $k_z=m\pi/a$, where the integer index $m$ gives the number of half waves along $z$. The angular dependence has to be expressed in an exponential form as $exp(\pm jn\theta)$, where the integer $n$ gives the number of waves azimuthally [2]. The radial dependence of the fields is expressed in the air by the Hankel function $H_n^{(2)}(k_{to}r)$ describing an outgoing attenuated wave (the transverse wavenumber $k_{to}$ in the air ($r>R$) has to be purely imaginary: $k_{to}=-j\alpha_{to}$). In the ferrite medium ($r<R$), the radial dependence can be given in terms of linear combination of the Bessel function $J_n(k_{t1}r)$ and $J_n(k_{t2}r)$ with different transverse wavenumbers $k_{t1},k_{t2}$ [6].

By enforcing the continuity of the tangential components $E_z,H_z,E_\theta,H_\theta$ at the interface ($r=R$), a $4x4$ coefficient matrix is obtained; the zeroing of the determinant gives a characteristic eigenvalue equation to be solved numerically (for the sake of brevity, all the cumbersome analytical manipulations are not reported here). Once both the angular and the longitudinal variations are fixed (through $n$ and $m$, respectively), the eigen-solutions can be ordered radially with an integer index $p$. The calculation of the modal resonant frequencies, of the related wavenumbers, and then of the field configurations allows the complete characterization of the FRs.

## 3. Lossless case: complete resonance mode spectrum

The complete spectrum of resonant frequencies has been calculated accurately, as a function of the involved electromagnetic and geometric parameters, by means of standard procedures for the lossless case (where the eigenvalues are real). The usual modal classification in such structures can be derived by a suitable extension of the classification in the isotropic problem [7], in terms of hybrid electromagnetic modes (HEM) (or in terms of HE and EH modes [1,8]). In our case, for solutions without rotational symmetry ($n\neq 0$), the effect of anisotropy is described by pairs of nonreciprocal solutions with a same choice of the angular, radial, longitudinal indices $(n,p,m)$ but with different angular phase variations, one counter-clockwise ('+' sign for $n$), the other clockwise ('−' sign for $n$); it can be noted that, by inverting the direction of the static magnetic field, these modes interchange their role. Therefore, we can generally classify such FR modes as $HEM^{\pm}_{npm}$.

Various types of mode spectra for the resonance frequencies are then obtainable for fixed choices of the physical parameters. In particular, considering the modal frequency distribution for fixed angular and axial dependences and varying $p$, an interesting 'clustering' phenomenon may occur, that is typical of lossless structures when the frequency for which $\mu_1=0$ is approached: such 'critical frequency' $f_c$, where the resonances are crowded, is related to both the magnetization characteristic of the material and the applied static magnetic-field intensity (in terms of angular frequencies, it is: $\omega_c=2\pi f_c=[\omega_o(\omega_o+\omega_m)]^{1/2}$). It can be seen that by changing such parameters, this effect of resonance thickening can deeply be varied.

A plot of resonance modes as a function of the magnetization parameters is presented in Fig. 2, where values proportional to resonance frequencies for the first modes are expressed as a function of $|\mu_2/\mu_1|$ (we should note that such a choice of variables is not immediately significant from a physical point of view, since $\mu_1$ and $\mu_2$ depend on frequency as well). Actually, such choice of parameters has been selected to reach direct comparisons with one of the few cases in the literature that can approximately be reduced to that one of our interest (see, e.g., Fig. 4 of [8]). Solutions presented in [8] were obtained with fully numerical techniques for an anisotropic resonator inside a cylindrical metallic cavity: when the lateral boxing is at least at a distance twice the FR radius, then the results for NRD case can approximately be compared. Generally, our data have been demonstrated in good agreement with reliable reference data achievable from the literature.

It should also be reminded that, in NRD structures, a typical $m=1$ vertical dependence is generally assumed [2,3], so that this class of modes has here a predominant practical interest (moreover, in our structure the $m=0$ solutions do not represent confined modes).

In NRD operations, the possible resonance frequencies for dielectric resonators are usually chosen inside a certain limited range [7]: the upper limit, $f_{max}$, is related to the maximum value to have an evanescent field in the air (i.e., suppression of radiation for $f<f_r=c/2a$); the lower limit, $f_{min}$, is commonly related to the minimum value to have a stationary field in the dielectric.

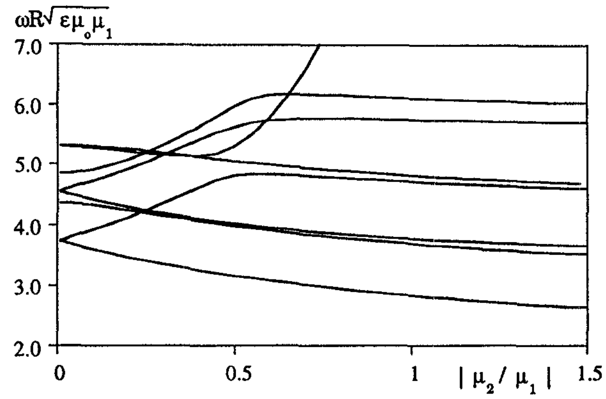



**Fig. 2** - Mode chart of normalized resonance values $\omega R\sqrt{\varepsilon\mu_o\mu_1}$ vs. $|\mu_2/\mu_1|$ to be compared with Fig. 4 of [8] for an analogous laterally-boxed structure. *Parameters: $R=a$, $\varepsilon_r=10$.*

The results of our analysis show that for NRD FRs, additional disregarded modal solutions (obtainable, e.g., for $n=\pm 1$, $m=1$, and $p=0$) are anyway possible below the usual frequency limit $f_{min}$, which correspond to a pair of evanescent surface waves on the interface (for this case both the ferrite and the air wavenumbers are imaginary).

Once the eigenvalue problem is solved, it is also possible to easily derive for any resonance the characteristic modal-field behaviors using closed-form expressions.

The important capability to tune the selective frequencies as the bias magnetization is varied will be discussed subsequently (Sect. 5), in connection with the experimental evidence of this phenomenon.

## 4. Loss effects in the ferrite and in the conductors

The evaluation of loss effects is particularly important to suitably model the effective practical behavior of ferrite components. In fact, it should be emphasized that the introduction of losses in ferrites can deeply modify the theoretically predictable behaviors: in fact, particular phenomena of the ideal lossless structures (e.g., clustering) disappear when realistic physical parameters are employed.

If dissipation is taken into account in ferrites according to the well-known Landau-Lifshitz approach [1], the frequencies become generally complex ($f=f_R+jf_I$) since an additional imaginary part of the resonance frequency takes into account damping oscillations. Thus, to have a quantification of these loss effects in NRD-disk FRs, the already-described characteristic eigenvalue equation has here accurately been solved with specific routines for finding solutions on the complex plane.

As concerns the influence of losses to quantify the unloaded quality factors $Q_u$ for the various resonance modes, it should be noted that in NRD resonators the possible contributions may be related to the losses in the material and on the conductor planes, while radiation loss is suppressed due to the nonradiative nature of these structures [2,7]. Therefore, we have: $1/Q_u = 1/Q_f + 1/Q_c$, where $Q_f$ is the Q-contribution due to a non-zero damping factor in the ferrite and $Q_c$ is the Q-contribution due to a finite conductivity in the parallel metal plates. It should be noted that for calculating both $Q_f$ and $Q_c$ of any mode, analytical procedures have been carried out here (the derivations cannot be described due to space limits).

Accurate evaluations of the unloaded quality factor have therefore been possible. An example of the behavior of quality factors due to both ferrite $Q_f$ and conductor $Q_c$ is shown in Figs. 3a and 3b, respectively, as a function of the NRD-FR radius, for the first three clockwise modes with $n=1$ and $m=1$. Generally, the ferrite Q-factors decrease as the radial modal index increases.

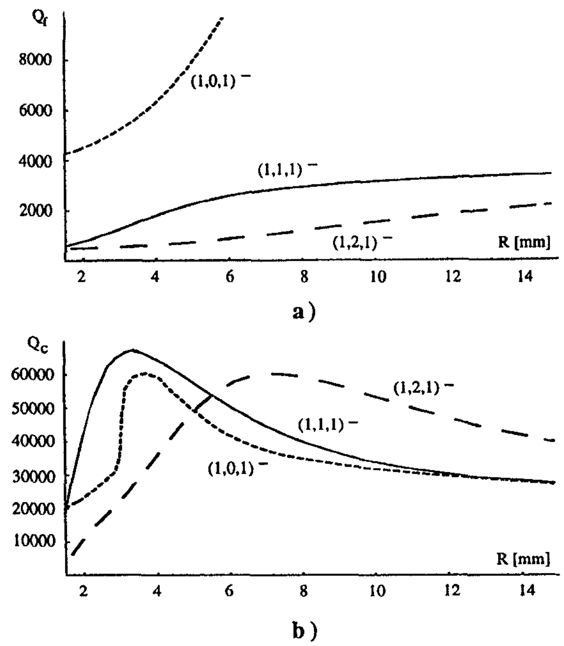



**Fig. 3** - Effect of ferrite and conductor losses in the unloaded quality factor $Q_u$: **a)** ferrite Q-factor $Q_f$ vs. $R$ for the first three $n=1$, $m=1$ clockwise modes; **b)** as in a) for the conductor Q-factor $Q_c$. *Parameters: $\alpha=10^{-3}$; $\sigma=6.17 \cdot 10^7$ S/m; $H_o=6000$ Oe; the rest as in Fig. 1.*

## 5. Experimental investigation with prototypes of NRD ferrite components

An experimental setup has been designed and assembled to test the accuracy of the theoretical analysis and verify the practical performances of such NRD frequency selective components.

As basic feeding structure, we have realized an NRD waveguide with a rod in rexolite and with silvered boxing plates sufficiently wide to avoid radiation. For ease of manufacturing, the components have been designed for testing in $X$ band, particularly in the *10-12 GHz* range [7].

An electromagnet has been used for biasing the components, with magnetization fields variable continuously between around *1500* and *8000 Oe*; such range is related to the stabilized current source that furnishes $DC$ values between around *2* and *12 A*.

All the measurements have been performed with a vector network analyzer; to control the magnetic field intensity a Hall-effect gaussmeter has also been used.

Samples of resonators in ferrite have been employed, usually coupled to the feeding line in a simple bandstop configuration. Nonreciprocal effects can be emphasized by measuring the excitation of resonant modes with a same index pair (either clockwise or counter-clockwise): different selective behaviors can be noted for the direct and reverse transmission scattering parameter (i.e., $S_{21}$ and $S_{12}$), or for the positive and negative magnetization directions. It should be noted that the amount of excitation of the resonance modes (location, amplitude, and width of the absorption peaks) is particularly affected by the coupling distance between guide and FR, but depends in a complex way also on other physical quantities [4,7].

The important effects of tuning as bias is varied have then been tested theoretically and experimentally. A representative chart of calculated resonance frequencies vs. bias magnetization is reported in Fig. 4a for two pairs of modes with different radial variations. In general, the other parameters being equal, the amount of frequency separation for a couple of same-index resonances is strongly dependent on the bias intensity and on the mode type as well. A good degree of agreement between theoretical and experimental data for these characteristic behaviors has been achieved, as shown in Fig. 4b for the tuning curves of a selected mode. Relative errors are generally limited by few percents.

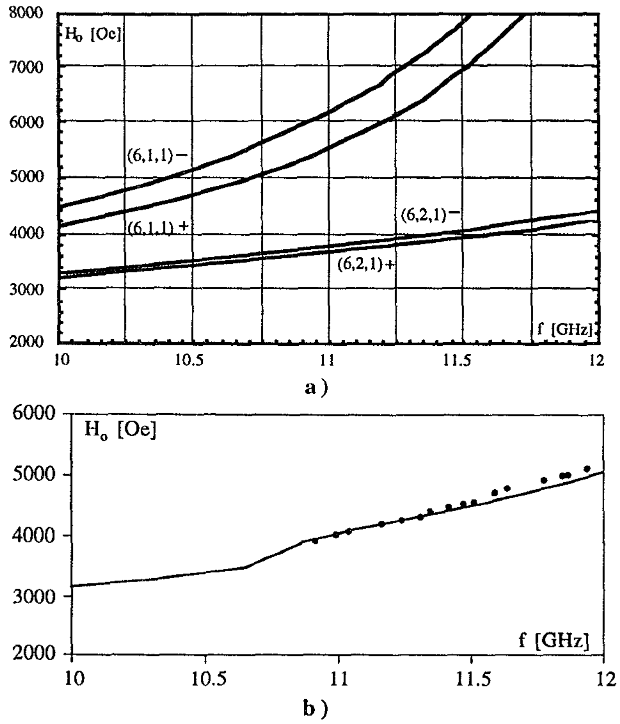



**Fig. 4** - Tuning effects of bias magnetization $H_o$ on the resonances $f$: **a)** theoretical values for the $n=6$ first modes; **b)** comparisons of experimental data (dots) with theoretical ones (solid line) for the $(1,6,1^+)$ mode. *Parameters: as in Fig. 1.*

Measurements have also been made on the quality factors. Since no precise information was available on the ferrite dissipation behavior of our samples, by knowing the conductor losses it has been possible to evaluate experimentally the ferrite losses, thus quantifying usefully the damping factor.

In all these comparisons, we have carefully considered and evaluated also the effects of geometric demagnetization [1].

## 6. Conclusion

The fundamental theoretical knowledge concerning the use of biased cylindrical ferrite resonators for integrated NRD components, such as circulators and tuning filters, has been presented here. Using accurate methods, a complete characterization of the resonance modes has been achieved as a function of the various parameters involved, concerning the frequency spectrum, the field behavior, the loss effects in ferrite and conductors, the quality factors, and so on. The nonreciprocal selective properties and the tuning capabilities have been tested also experimentally with measurements on prototypes of ferrite resonators integrated in NRD guide.

Useful information is thus achieved for accurate design of various types of nonreciprocal components, which further enriches the potentialities both of ferrites and of NRD circuitry for millimeter-wave advanced applications.

## References


[1] B. Lax and K. J. Button, *Microwave ferrites and ferrimagnetics*. New York, NY: McGraw-Hill, 1962.

[2] T. Yoneyama, "Nonradiative dielectric waveguide," in K. J. Button, Ed., *Infrared and Millimeter-Waves*. New York, NY: Academic Press, vol. 11, pp. 61-98, 1984.

[3] C. Di Nallo, F. Frezza, A. Galli, P. Lampariello, and A. A. Oliner, "Properties of NRD-guide and H-guide higher-order modes: physical and nonphysical ranges," *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 12, pp. 2429-2434, Dec. 1994.

[4] F. Frezza, A. Galli, G. Gerosa, and P. Lampariello, "Characterization of resonant and coupling parameters of dielectric resonators for NRD filtering devices," *IEEE MTT-S Int. Microwave Symp. Dig.*, pp. 893-896, 1993.

[5] H. Yoshinaga and T. Yoneyama, "Design and fabrication of a nonradiative dielectric waveguide circulator," *IEEE Trans. Microwave Theory Tech.*, vol. 36, no. 11, pp. 1526-1529, Nov. 1988.

[6] M. L. Kales, "Modes in wave guides containing ferrites," *J. Applied Physics*, vol. 24, no. 5, pp. 604-608, May 1953.

[7] F. Frezza, A. Galli, G. Gerosa, and P. Lampariello, "Resonant frequencies and quality factors in lossy NRD cylindrical resonators," *Int. J. Inf. Millimeter Waves*, vol. 16, pp. 675-688, Mar. 1995.

[8] J. Krupka, "Resonant modes in shielded cylindrical ferrite and single-crystal dielectric resonators," *IEEE Trans. Microwave Theory Tech.*, vol. 37, no. 4, pp. 691-697, Apr. 1989.